

Optimal sensor placement for structural health monitoring based on multiple optimization strategies

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SUMMARY

Careful selection and placement of sensors are the critical issue in the construction and implementation of an effective structural health monitoring system. A hybrid method termed the optimal sensor placement strategy (OSPS) based on multiple optimization methods is proposed in this paper. The initial sensor placement is firstly obtained by the QR factorization. Then, using the minimization of the off-diagonal elements in the modal assurance criterion matrix as a measure of the utility of a sensor configuration, the quantity of the sensors is determined by the forward and backward sequential sensor placement algorithm together. Finally, the locations of the sensor are determined by the dual-structure coding-based generalized genetic algorithm (GGA). Taking the scientific calculation software MATLAB (MathWorks, Natick, MA, USA) as a platform, an OSPS toolbox, which is working as a black box, is developed based on the command-line compiling and graphical user interface-aided graphical interface design. The characteristic and operation method of the toolbox are introduced in detail, and the scheme selection of the OSP is carried out on the world's tallest TV tower (Guangzhou New TV Tower) based on the developed toolbox. The results indicate that the proposed method is effective and the software package has a friendly interface, plenty of functions, good expansibility and is easy to operate, which can be easily applied in practical engineering. Copyright © 2011 John Wiley & Sons, Ltd.

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1. INTRODUCTION

With the extensive utilization of large-scale structures, such as high-rise buildings, suspension bridges and transmission towers, in various engineering applications, the structural health monitoring (SHM) for these types of structures has been the topic of many research efforts. The SHM could provide vital information for the safe operation of civil structures and enables operational cost reduction by performing prognostic and preventative maintenance (Li *et al.*, 2004). It shows also a great potential for disaster mitigation, for instance, the catastrophic roof collapse of Terminal 2E at the Paris Charles de Gaulle Airport. Such collapses or failures of critical civil infrastructures stimulate the increasing concerns about structural integrity, durability and reliability, i.e., the health state of a structure. On the other hand, advances in sensing technology have enabled the use of large numbers of sensors for the SHM. However, high cost of the data acquisition systems (sensors and their supporting instruments) and accessibility limitations constrain wide distribution of sensors in many cases. Especially, many structures have to be tested under operational conditions for online health monitoring, in which the sensors are not easily amenable to be removed or relocated because the communication cables and even the sensors were buried in the structure. Furthermore, structural responses measured at specified sensor positions determine the accuracy of modal

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parameter identification and are crucial in the consequent model updating and damage quantification. Therefore, how do optimally determine the number of sensors and deploy them so that the response data acquired from those locations will result in the best identification of structural characteristics are challenging tasks (Kistera *et al.*, 2007; Majumder *et al.*, 2008; Wenzel, 2009).

Normally, the sensor selection is based purely on the civil engineers' judgment. For a structure that has simple geometry, or smaller number of degrees of freedom (DOF), the experience and a trial-and-error approach may suffice to solve the problem. For a large-scale complicated structure, whose finite element (FE) model may have tens of thousands of DOFs, a systematic, reliable and efficient approach is needed to solve such a computationally demanding problem. Numerous techniques have been advanced for solving the optimal sensor placement (OSP) problem and are widely reported in the literature. One of the most significant and commonly cited OSP approaches used for structural monitoring was developed by Kammer in 1991. In his study, Kammer argued that the optimal arrangement for measuring and estimating the structural vibration was that which minimized the norm of the Fisher information matrix (FIM), which was constructed from the modal and measurement covariance matrices. Salama *et al.* (1987) proposed using modal kinetic energy (MKE) as a means of ranking the importance of candidate sensor locations. Moreover, there had been several other variants on this theme, such as average kinetic energy and weighted average kinetic energy proposed by Chung and Moore (1993). Li *et al.* (2007a, 2007b) studied the relation between the effective independence (EI) Method and the MKE Method. Moreover, maximizing the area of coverage per sensor using geometrical and physical constraints was suggested in a few methods (Lin and Chiu, 2005). Others focused on enhancing the detection efficiency and minimizing uncertainty in decision making based on the data acquired from the sensor networks (Field and Grogoriu, 2006). Chang and Markmiller (2006), as a measurement for quantifying the reliability of a sensor network, defined the probability of detection (POD). The optimal sensor network was introduced as the network sensor configuration that could achieve the target POD. Liu and Tasker (1996) presented a perturbation-based approach to predict the optimal locations for sensors. Heredia-Zavoni *et al.* (1999) treat the case of large model uncertainties expected in model updating. The optimal sensor configuration was chosen as the one that minimized the expected Bayesian loss function involving the trace of the inverse of the FIM for each model. Furthermore, Meo and Zumpano (2005) compared the capabilities of various methods by assessing the results of their use in the modal identification of a bridge. Far too many sensor placement approaches exist to mention them all in this paper. A thorough review of sensor placement methodologies concerning dynamic testing and criteria for judging the effectiveness of different methodologies can be found in Li *et al.* (2007a, 2007b).

As it follows from the literature review as mentioned earlier, many researchers have made their valuable contributions to the topic of effective sensor placement. To avoid difficulties, lots of researchers usually focus on finding the optimal locations for a given number of sensors. However, in a real application, the assumption that the number of sensors is known *a priori* is not a realistic and often an unpractical one. In addition, most engineers do not want to become computer programmers and write the tedious routines that are necessary to optimize the sensors and to display the final optimal results. Thereby, this paper was aimed at developing a method (called optimal sensor placement strategy (OSPS)) that may be used practically by civil engineers. To facilitate the use of the implemented method, a graphical user interface (GUI) that gives a user possibility to apply the method in a straightforward way and to visualize the results is developed.

The remaining part of the paper is organized as follows. Section 2 presents the hybrid algorithm used for the OSP based on multiple optimization strategies. Section 3 describes the developed MATLAB (MathWorks Inc., MA, USA) toolbox—OSPS, its organization, features and limitations. Section 4 shows the performance of the algorithm and the developed software package for the OSP in the world's tallest TV tower—Guangzhou New TV Tower (GNTVT). In Section 5, some overall conclusions are drawn, and trends of future work are stated.

2. BRIEF DESCRIPTION OF THE PROPOSED METHOD

The underlying idea of the OSP is to identify a sensor layout that will provide the best possible performance of the structures. For this reason, a step by step procedure, termed as the OSPS, based on

multiple optimization strategies is proposed here to determine the optimal number of sensors and the best strategy to deploy them in order to ensure the mode shapes, which are orthogonal.

It is known from the structural dynamic principle that the structural inherent modes should comprise a group of orthogonal vectors at the nodes. But in fact, it is impossible to guarantee that the measured modal vectors are orthogonal, because of the problems of the measured freedoms that are less than those of the model and measuring accuracy limitation. Further, it is even possible to lose many important modes owing to the too small space angles between vectors. The larger space angles among the measured modal vectors should be guaranteed while choosing measuring points in order to keep the original properties of the model if possible. Carne and Dohmann (1995) thought that the MAC was an ideal scalar constant relating the causal relationship between two modal vectors

$$\text{MAC}_{ij} = \frac{(\Phi_i^T \Phi_j)^2}{(\Phi_i^T \Phi_i)(\Phi_j^T \Phi_j)} = \frac{a_{ij}^2}{a_{ii}a_{jj}} \quad (1)$$

where, Φ_i and Φ_j represent the i th and the j th column vectors in matrix Φ , respectively, and the superscript T denotes the transpose of the vector.

In the above formulation, the element values of the MAC matrix range between 0 and 1, where 0 indicates that there is little or no correlation between the off-diagonal element $\text{MAC}_{ij}(i \neq j)$ (the modal vector easily distinguishable) and 1 indicates that there is a high degree of similarity between the modal vectors (the modal vector fairly indistinguishable). For an optimal (orthogonal) set, the MAC matrix would be diagonal; thus, the size of the off-diagonal elements could be an indication of optimal result. Therefore, the method used here is to initialize the selection of the sensor set with a small set of locations; then, a procedure, called sequential sensor placement (SSP), is used to add one sensor at a time, choosing from all the DOF available in the FE model in order to reduce the off-diagonal elements to an acceptable magnitude; finally, a global optimization algorithm is adopted to determine the final locations of the sensors.

2.1. Initial sensor assignments by QR factorization

The vector of the measured structural responses denoted by y_s , can be estimated as a combination of N mode shapes through the following expression:

$$y_s = \Phi q + \omega = \sum_{i=1}^N q_i \phi_i + \omega \quad (2)$$

where Φ is the matrix of FE model target mode shapes; q denotes the coefficient response vector; ω represents the sensor noise vector, which could be assumed stationary Gaussian white noise variance σ^2 ; N means the column number of Φ (n by N matrix, n being the number of the candidate sensor positions) and ϕ_i stands for the i th column of Φ that is the i th target mode shape selected.

This representation of structural response is based on the concept that, the response in any point of an elastic structure can be obtained in the time or frequency domain as a linear combination of mode shape values. In this way, the i th coefficient of y_s is a linear combination of the i th mode shape vectors, where q_i is a multiplier coefficient that is a function either of time or frequency. Evaluating the coefficient response vector using an efficient unbiased estimator and then estimating the covariance of the error result in the following:

$$P = E[(q - \hat{q})(q - \hat{q})^T] = \left[\frac{1}{\sigma^2} \Phi^T \Phi \right]^{-1} = \sigma^2 Q^{-1} \quad (3)$$

where E denotes the expected value, \hat{q} means the efficient unbiased estimator of q and Q is termed as the FIM.

Therefore, the procedure for selecting the best sensor placements is to unselect candidate sensor positions such that the determinant of the FIM is maximized. It is well known that maximizing the

determinant of Q approximately equals to maximize the norm of Q . Kammer (1991, 2004) suggested the spectral norm as a useful and physically meaningful matrix norm. By the norm theory, one could obtain the following expression:

$$\|Q\| = \|\Phi^T \Phi\| = \|\Phi^T\|_2^2 \quad (4)$$

Thereby, maximizing Φ_s^T would lead to the minimization of P and, thus, the best state estimate q . It is known that the QR factorization of a matrix is a decomposition of the matrix into an orthogonal matrix and a triangular matrix. Suppose that the subset of candidate location corresponding to the obtained mode matrix from the FE model be Φ , $\Phi \in R^{n \times N}$, and generally $N < n$ and $r(\Phi) = N$. By the QR factorization of the matrix Φ^T , the initial candidate set of sensor locations could be obtained as follows:

$$\Phi^T E = QR = Q \begin{bmatrix} R_{11} & \cdots & R_{1n} & \cdots & R_{1N} \\ & \ddots & \cdots & \cdots & \cdots \\ 0 & & R_{nn} & \cdots & R_{nN} \end{bmatrix} \quad (5)$$

where, $E \in R^{n \times n}$ is the permutation matrix; $Q \in R^{N \times N}$, $R \in R^{N \times n}$ and $|R_{11}| > |R_{22}| > \cdots |R_{NN}|$.

2.2. Determination of optimal number of sensors by sequential sensor placement algorithm

The computations involved in the SSP algorithm are an infinitesimal fraction of the ones involved in the exhaustive search method and may be done in realistic time, independence of the number of sensors and the number of model DOFs.

Let $\Phi(m \times N)$ and $\hat{\Phi}(\hat{m} \times N)$ denote the mode shape matrix for the initial selected sensor set and the remaining DOF, respectively, where m is the number of DOF in the existing sensor set, \hat{m} means the number of remaining DOF from which to select, and N implies the number of modes.

The positions of sensors are computed sequentially by placing one sensor at a time in the structure at a position that results in the highest reduction in the maximum off-diagonal element of the MAC. Specifically, the position of the first sensor is chosen as the one that gives the highest reduction in the maximum off-diagonal element for one sensor. When the row k of $\hat{\Phi}$ is added to Φ , then the MAC value between i and j with the added DOF k becomes

$$(\text{MAC}_{ij})_k = \frac{(a_{ij} + \hat{\Phi}_{ki} \hat{\Phi}_{kj})^2}{[a_{ii} + \hat{\Phi}_{ki}^2][a_{jj} + \hat{\Phi}_{kj}^2]} \quad (6)$$

After a sensor is added to the existing set, the matrixes Φ , $\hat{\Phi}$ and MAC should be updated to reflect this change. Given the optimal position of the first sensor, the position of the second sensor is chosen as one that gives the highest reduction in the maximum off-diagonal element computed for two sensors with the position of the first sensor fixed at the optimal one already computed in the first step. Continuing in a similar fashion, given the positions of $(s-1)$ sensors in the structure computed in the previous $(s-1)$ steps, the position of the next s th sensor is obtained as the one that gives the highest reduction in the maximum off-diagonal element for s sensors with the positions of the first $(s-1)$ sensors fixed at the optimal ones already obtained in the previous $(s-1)$ steps. This procedure is continued for up to N_0 sensors. Although the iterative nature of the optimization process only seeks a suboptimal or near-optimal solution, the result is believed to be close to the optimal one (Papadimitriou, 2004; Li *et al.*, 2008). For the sake of reference, the aforementioned algorithm is termed as the forward SSP (FSSP) algorithm.

The SSP algorithm can also be used in an inverse order, starting with N_d sensors placed at all DOFs of the structure and removing successively one sensor at a time from the position. This algorithm is termed as the backward SSP (BSSP). Accordingly, the MAC value between i and j with the deleted DOF k becomes

$$(\text{MAC}_{ij})_k = \frac{(a_{ij} - \hat{\Phi}_{ki} \hat{\Phi}_{kj})^2}{[a_{ii} - \hat{\Phi}_{ki}^2][a_{jj} - \hat{\Phi}_{kj}^2]} \quad (7)$$

The performance of the two methods depends on the number of sensors and observable modes. Thus, one may experience the non-decreasing aspect of these algorithms, i.e. the maximum off-diagonal term is not monotonically decreasing with the number of sensors. To alleviate the contra-decreasing problem of the algorithm, the combinational results provided by the SSP methods are recommended here to determine how many sensors are required.

2.3. Optimization of sensor locations by generalized genetic algorithm

Although the SSP method could also yield the sensor configuration, the derived sensor configuration result is only suboptimal because it is generated in an iterative manner. The genetic algorithm (GA) is a computational intelligence method to the OSP problem that tends to get the global optimum effectively. An artificial GA starts with a discrete set (also called generation) of design vectors (also called individuals, chromosomes or strings), and through three main operations (selection, crossover and mutation), it modifies the current set towards generating a better generation of design points (Holland, 1975). In every generation, the fittest chromosomes are given a greater chance of survival and also of passing their genes to the next generation. The GA has been proved to be a powerful tool to the OSP, but it also has some faults needing to be fixed. For example, when the GA is used to solve the OSP, the general crossover and mutation operators may generate chromosomes that do not satisfy the constraints. One location should not be placed with two or more sensors, or sensor number is equal to a certain number. Therefore, some improvements are proposed to overcome these faults. Here, an improved GA called generalized genetic algorithm (GGA) is adopted to find the optimal location of sensors. The GGA is based on some famous modern biologics theories such as the genetic theory by Morgan, the punctuated equilibrium theory by Eldridge and Gould, and the general system theory by Bertalanffy, so it is superior in biologics to classical GA (Dong, 1998). For the sake of completeness, a brief discussion of the GGA is given here. For more detail, the reader is referred to the related references.

The GGA could be described as applying the crossover on the two parents selected from the initial population; remaining two individuals with best fitness values from the 4 individuals, of which two before the crossover and two created after the crossover (i.e. two quarters selection); eliminating the two individuals with the worst fitness values; applying the mutation on the two remains and using two quarters selection on the four individuals according to the fitness values; and finally, the next generation is produced by $N/2$ times repeating the above steps.

The difference between the GGA and simple genetic algorithm (SGA) mainly reflects in the evolutionary process. In short, the evolutionary process of the SGA is

Two parents selection → crossover → mutation → survival selection → next generation

Whereas the process of the GGA is

Two parents selection → crossover → a family of four → two quarters selection → mutation → a family of four → two quarters selection → next generation

It can be found that the two quarters selection is introduced in the GGA. The parents are allowed to compete with the children during the process of crossover and mutation and only the best one may enter next competition. In addition, the crossover and mutation of the GGA have little difference with the SGA. In the SGA, the crossover may operate according to a certain probability, whereas for the GGA, because the parents are certain to join in the competition, the crossover probability remains as 1.

The evolutionary process of the GGA has two stages: gradual change and sudden change. For the gradual change, the local optimum may be achieved mainly by the crossover and selection, in which the single-point crossover and swap mutation are generally used and the sequence of operations is first crossover and then mutation. For the sudden change, the global optimum could be reached mainly by mutation and selection, which realize the escape from one local optimum to better local optimum, in which the uniform crossover and inversion mutation are mainly used and the sequence of operations is first mutation and then crossover. The gradual change will be automatically transferred to the sudden change by feedback element when the evolutionary process tends to local convergence; as well, once the leader of population is changed, the sudden change may be automatically transferred to the gradual change by the feedback element, too.

From the view of mathematics, the OSP is a kind of particular knapsack approach, which places specified sensors at optimal locations to acquire more structural information. Its mathematic model is a 0–1 programming problem if the value of the j th gene code is 1; it denotes that a sensor is located on the j th DOF. In contrast, if the value of the j th gene code is 0, it indicates that no sensor is placed on the j th DOF. The total number of 1 in a chromosome is equal to the sensor number. Most commonly, the design variables are coded by the simple one-dimension binary coding method that is very simple and intuitionistic. However, the number of 1 will be changed in the crossover and the mutation (i.e. the number of sensors will be changed), which cannot meet the demand of the OSP. Here, a dual-structure coding GA is adopted to overcome this problem.

The dual-structure coding method is shown in Table 1. The chromosome of individual is composed of two row, in which the upper row $s(i)$ denotes the append code of x_j , and $s(i)=j$, the lower row represents the variable code $x_{s(i)}$ corresponding to the append $s(i)$. When coding certain individuals, the shuffle method is firstly used to produce stochastic $\{s(i), (i=1, 2, \dots, n)\}$ and list on the upper row; then, the variable code (0 or 1) is generated randomly. For example, an OSP problem has 10 sensors, and the randomly generated order of append code is (4, 3, 5, 8, 6, 10, 2, 7, 9, 1). Thus the feasible solution, namely the sensors are located on the third, fifth, sixth, seventh and first DOFs. This kind of the method only operates on the upper append code, and the lower variable value of offspring is fixed. That means that the number of sensors could be unchanged.

3. THE DEVELOPMENT OF THE OSPS TOOLBOX

To facilitate the use of the presented method, a flexible toolbox termed the ‘OSPS Toolbox’ that gives a user possibility to apply the method in a straightforward way and to visualize the results was developed within the MATLAB programming environment (2008). MATLAB has been chosen as a platform for the OSPS software development as it provides a technical computing environment that combines numeric computation, visualization and higher-level programming language. The interactive MATLAB workspace provides a flexible interface between the user and the computer memory, which allows direct access to all variables that are created during a session. Its built-in functions and graphics capabilities provide a useful environment for testing new algorithms. The MATLAB was also designed to allow easy extensibility. The Code developed in MALTLAB is portable between operating systems (e.g. UNIX, PC and Macintosh). More specialized routines are available through additional toolboxes or could be programmed by the user.

3.1. The characteristic of the OSPS Toolbox

The OSPS Toolbox is designed in a modular fashion and is therefore highly flexible in terms of portability and extendibility. In order to achieve maximum flexibility for the user, the OSPS has been

Table 1. Dual-structure coding method.

Append code	$s(1)$	$s(2)$	$s(3)$...	$s(i)$...	$s(n)$
Variable code	$x_{s(1)}$	$x_{s(2)}$	$x_{s(3)}$...	$x_{s(i)}$...	$x_{s(n)}$

written mainly in the form of scripts. These allows for the application of user-defined routines at any time, to make immediate changes to the code without the need for recompiling and, if needed, direct access to OSPS-internal variables during processing. Figure 1 schematically illustrates the functional block diagram of the OSPS showing the main computational blocks and the algorithmic options. Different routines of the tools and helping demonstrations can be invoked either from the GUI environment of MATLAB command window.

The toolbox achieves what-you-see-is-what-you-get behavior in the MATLAB. A simple GUI is used to navigate the toolbox and thus allows a user to apply the implemented methods in an easy way (almost all the processing features could be accessed from the menu bar and control buttons), and it gives a straightforward possibility to visualize the obtained results. The GUI consists of a main window from which all the optimal methods could be easily accessed from here, as shown in Figure 2. To initialize the graphical interface, the current directory of MATLAB should be changed to the directory where the graphical interface is installed. The GUI program is initiated in the command window of MATLAB with the 'OSPS' command. FSSP, BSSP and GGA are among the implemented methods, as shown in Figure 3.

The graphical interface of each method is composed of an upper window menu and three panels: the figure panel (left), the parameter input panel (top-right), the result panel (bottom-right). A menu bar with five icon buttons at the top of the main window offers direct access to specific functionalities, namely, 'File', 'Edit', 'Tools', 'Window' and 'Help'. The 'File' menu is mainly responsible for loading mode data and saving calculation results. To load the data into the graphical interface, a user can easily select from the 'File' and chose the location of the data 'Import data from *.mat file' or 'Import data from workspace'. The last option is available only if the data are present in the MATLAB workspace. When the file or the workspace contains several variables, then the user is asked to select variables that should be imported to the graphical interface. Similarly, a user can also easily export the obtained results either to a 'mat' file or to the MATLAB workspace. This gives a possibility to apply other methods that are not available in the graphical interface on the saved or exported data, or to create custom plots. The 'Edit' menu functions allow changing the figure properties. The 'Tools' menu is available to enhance plots with zooming, panning, rotation, color maps, lighting and shading options. Shortcuts to routines and validation procedures have been grouped under the 'Window' menu. The last menu button is for launching the 'User's Guide' in a PDF format and for opening a dialog window displaying the program's version, respectively.

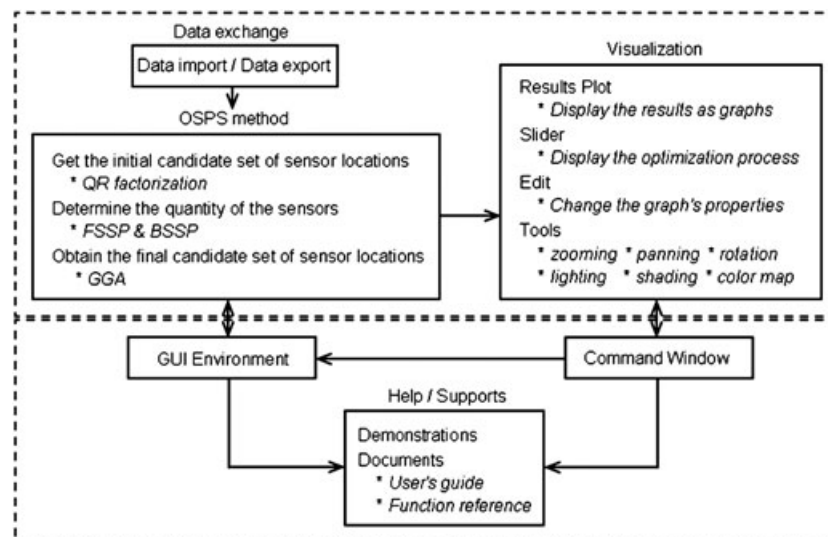


Figure 1. Functional block diagram of optimal sensor placement strategy (OSPS) showing the main computational blocks and the algorithmic options. BSSP, backward sequential sensor placement; FSSP, forward sequential sensor placement; GGA, generalized genetic algorithm; GUI, graphical user interface.

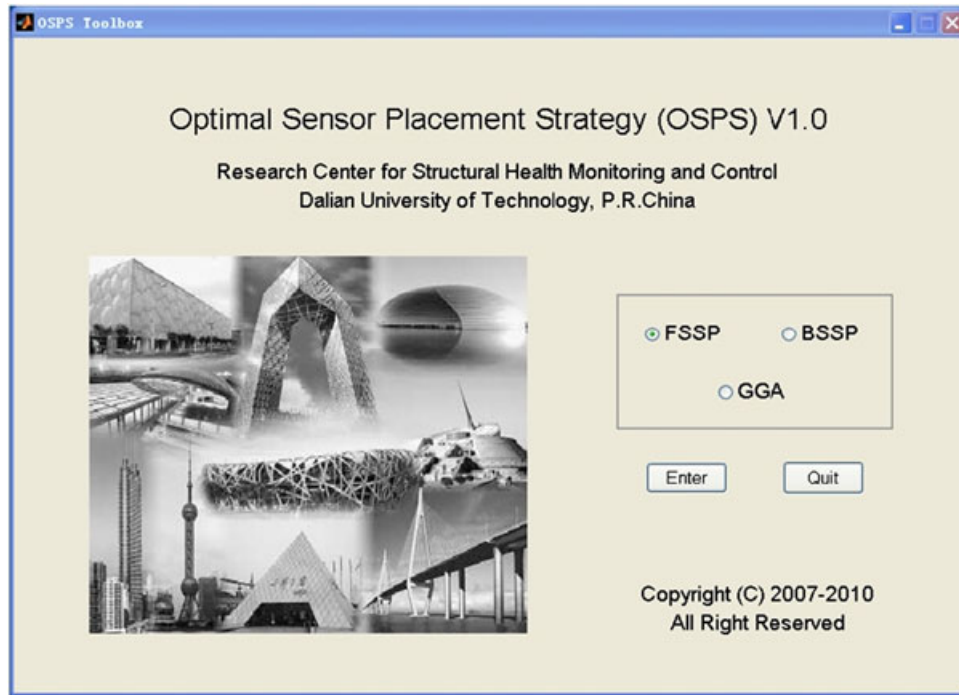


Figure 2. Main window of the graphical user interface of the OSPS Toolbox. BSSP, backward sequential sensor placement; FSSP, forward sequential sensor placement; GGA, generalized genetic algorithm.

In parameter input panel, with text boxes, the users can easily change some basic parameters (population size, number of sudden change, initial candidate set of sensor locations, etc.) needed prior to the optimization. The result panel is used to export the optimization results, such as run time, final locations of the sensors. It is worth mentioning that, in the graphical interface of the FSSP, BSSP and GGA, there is a slider. The different results can be visualized when changing the locations the slider. There are three function buttons in the bottom-right section of the interface, namely, 'Run', 'Close' and 'Quit', stand for launch the program, close the program and erase the variables from the graphical interface, respectively.

3.2. Software specifications and requirements

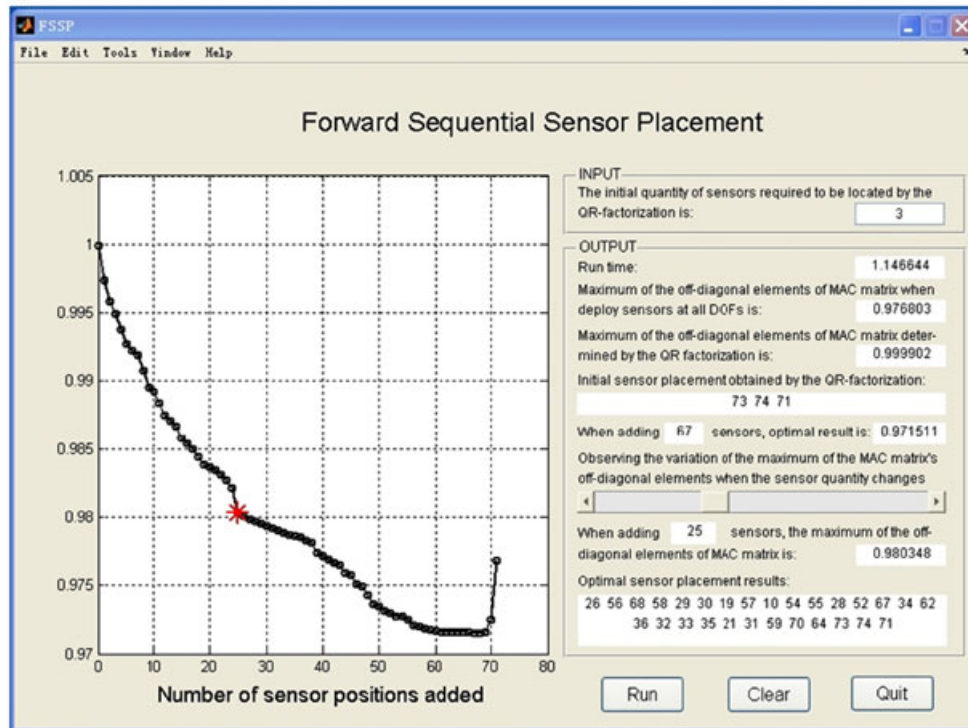
The OSPS is implemented with MATLAB 6.5 release 13 and tested with different versions of the MATLAB 7 on a Windows XP system, without requiring any other third parties, but the software should also run on other operating systems and the MATLAB versions. In fact, the toolbox provides all that is needed to get started without requiring much knowledge of software or hardware design. By providing a community-based platform, one wish to extend the scope of the toolbox and give rise to a reliable and free set of tools that are independent of software licenses or the doings of a specific company in the near future.

4. CASE STUDY

4.1. Guangzhou New TV Tower

The Guangzhou New TV Tower (GNTVT), currently being constructed in Guangzhou, China, when completed in 2010, will be the world's tallest TV tower with a total height of 610 m (Ni *et al.*, 2009). It consists of a main tower (454 m) and an antennary mast (156 m). As shown in Figure 4, this structure comprises a reinforced concrete interior tube and a steel external tube. The external tube consists of 24 concrete-filled-tube columns uniformly spaced in an oval configuration and inclined with respect to the vertical direction. The oval cross-section of the outer tube varies along the height of

a



b

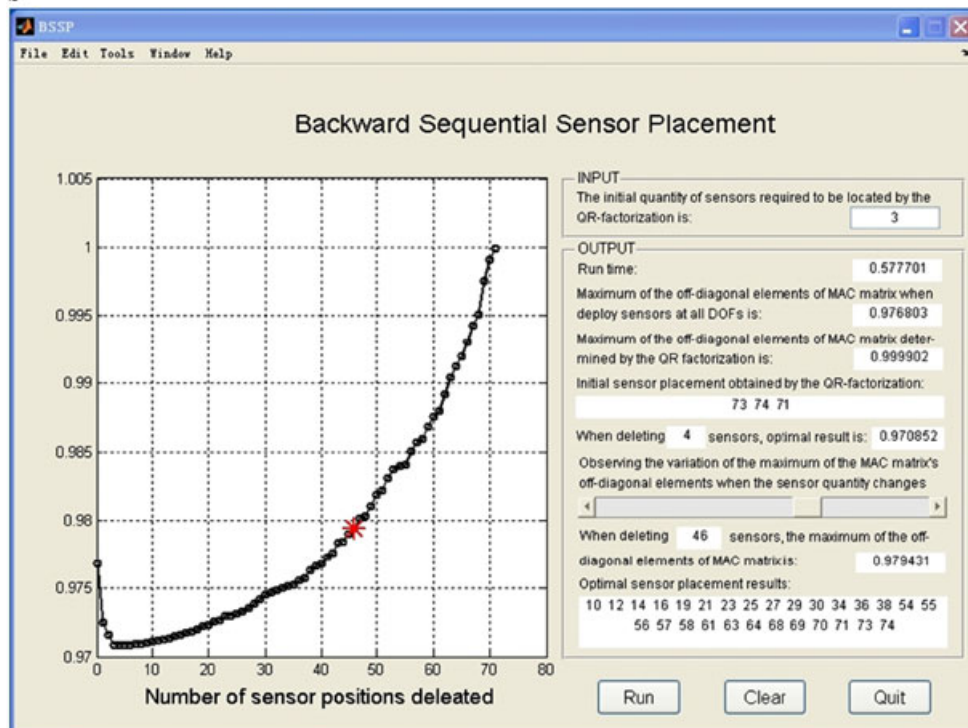


Figure 3. Interface of different algorithm contained in the toolbox: (a) forward sequential sensor placement; (b) backward sequential sensor placement; (c) generalized genetic algorithm.

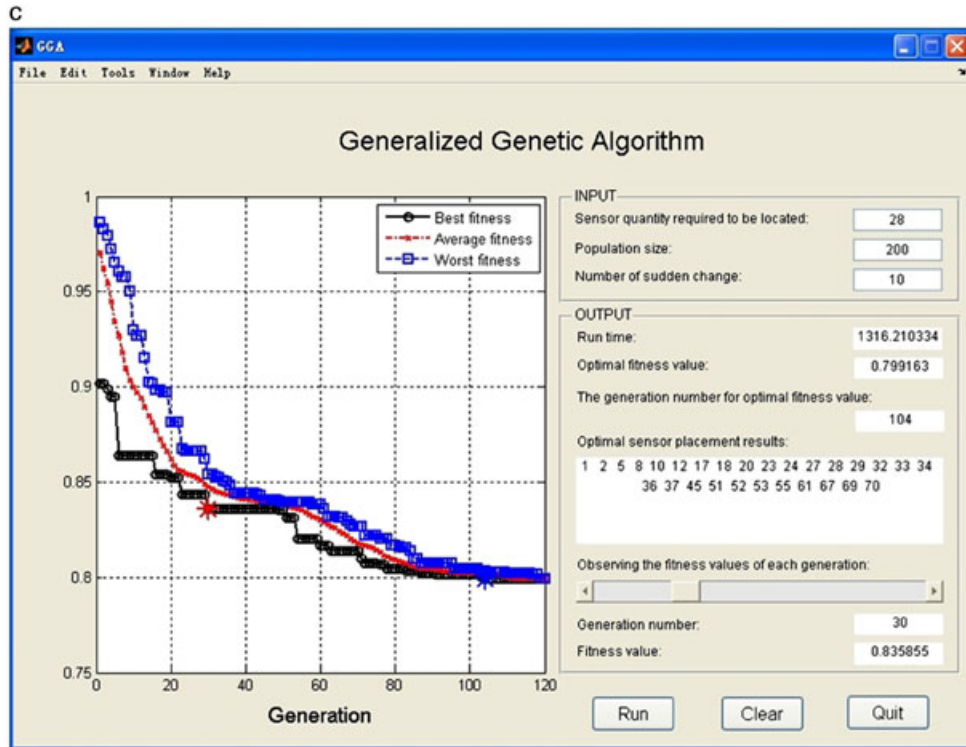


Figure 3. (Continued)

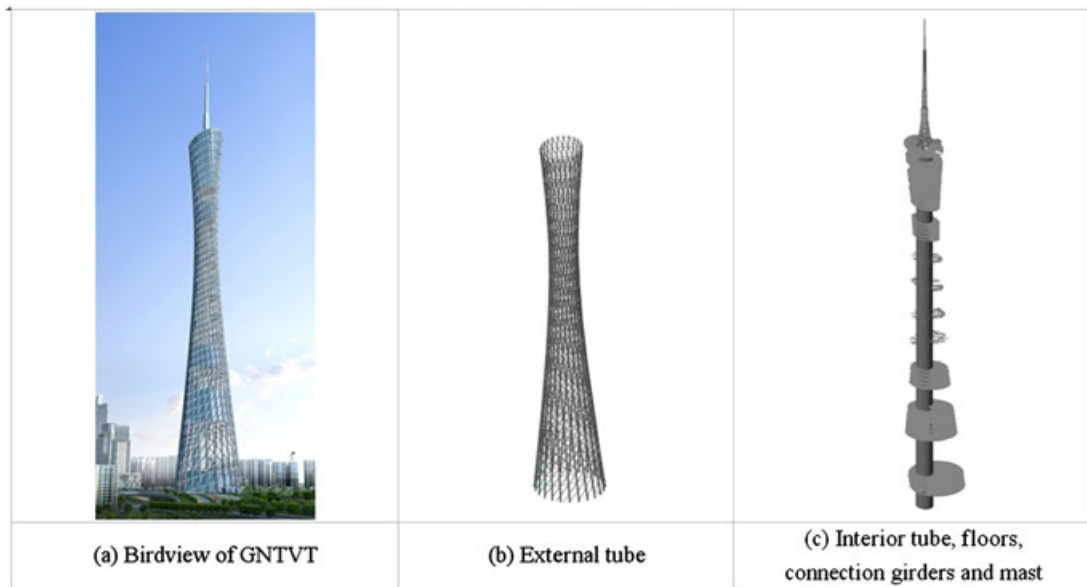


Figure 4. Guangzhou New TV Tower (GNTVT).

the tower, and it decreases from $50\text{ m} \times 80\text{ m}$ at ground level to the minimum dimensions of $20.65\text{ m} \times 27.5\text{ m}$ at the height of 280 m, and then increases to $41\text{ m} \times 55\text{ m}$ at the top of the tube (454 m). The interior tube is also an oval with constant dimensions of $14\text{ m} \times 17\text{ m}$, although the centroid differs from that of the external oval. This hyperbolic shape makes the structure interesting and attractive from an esthetics perspective, and it also makes it mechanically complex.

Civil engineering structures, particularly high-rise structures (like the GNTVT) are especially susceptible to random vibrations, whether it is from large ground accelerations, high wind forces and/or abnormal loads such as explosions. In order to enhance the safety of the GNTVT, researchers have made use of significant technological advances (such as wind tunnel test and shaking table test) in various disciplines of civil engineering (Pan *et al.*, 2008; Gu *et al.*, 2009; Tan *et al.*, 2009; Liu *et al.*, 2009; Guo *et al.*, 2010). Although there are lots of attempts to enhance the properties of the GNTVT, generally, these have led to improvements in the safe operation and preventative maintenance of the structure only in a certain extent. This is because the GNTVT will serve for a long period. During the service time, it is inevitable to suffer from environmental corrosion, material aging, fatigue and coupling effects with the long-term load and extreme load. The induced damage accumulation and performance degeneration due to the above factors would inevitably reduce the resisting capacity of the structure against disaster actions, even results in collapse with the structural failure under extreme loads. Therefore, to build an SHM system for the GNTVT is a necessary requirement. One crucial issue in construction and implementation of an effective SHM system in the GNTVT is how to determine the quantities of the sensors and select sensor locations from a set of possible candidate positions.

4.2. Numerical model for GNTVT

For a structure that has simple geometry, or smaller number of DOF, an experience and a trial-and-error approach may suffice to solve the problem. For a large-scale complicated structure like the GNTVT, whose FE model may have tens of thousands of DOFs, a systematic and efficient approach is needed to exclude the constrained node where the sensors cannot deploy. Here, the model simplification method and model reduction method was used to solve this problem.

4.2.1. Full-order FE model

In order to accurately replicate the behavior of the real structure, a fine three-dimensional (3D) FE model of the GNTVT constructed by Lin *et al.* (2010) using the ANSYS (ANSYS Inc., Canonsburg, PA, USA) software is applied, as shown in Figure 5(a). The 3D full-order model contains 122 476 elements, 84 370 nodes and 505 164 DOFs in total. In the model, PIPE16 and BEAM44 (2-node 3D beam elements with six DOFs at each node) are employed to model the outer structure, antenna mast and the connection girders between the inner and outer structures. Four-node and three-node shell

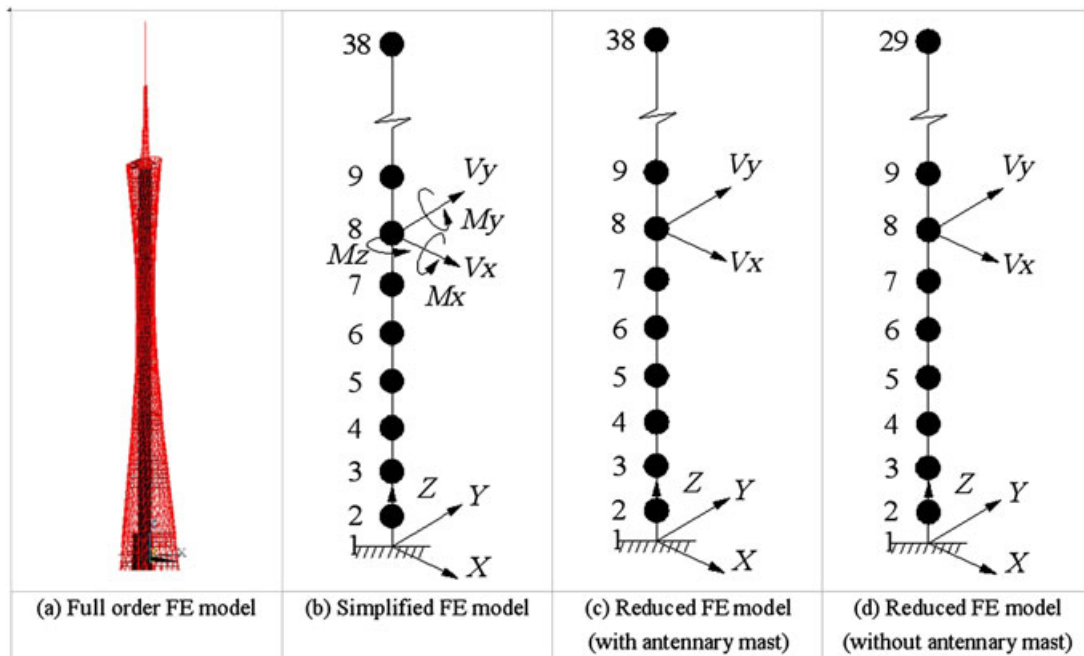


Figure 5. Numerical model for Guangzhou New TV Tower. FE, finite element.

elements with six DOFs at each node are used to model the shear walls of the inner structure and the floor decks.

4.2.2. Simplified FE model

The size of the 3D full-order model is too large for the OSP and related studies. Based on the full-order model, a simplified 3D beam model is established. Three assumptions are made in developing the simplified 3D beam model: (a) the floor systems are assumed as rigid bodies, (b) each segment between two adjacent floors is modeled as a linear elastic beam element, and (c) the masses are lumped at the corresponding floors. Consequently, the whole structure is modeled as a cantilever beam with 37 beam elements and 38 nodes. The main tower consists of 27 elements, and the mast is modeled as 10 elements. As shown in Figure 5(b), the nodal number increases from 1 at the fixed base to 38 at the free top end. As the geometric centroid of each floor varies, the beam of the simplified model aligns with the centroidal axis of the tower mast. The vertical displacement is disregarded in the simplified model, and thus each node has five DOFs, i.e. two horizontal translational DOFs and three rotational DOFs. As a result, each element has 10 DOFs, and the entire model has 185 DOFs in total. The floor mass is added to the corresponding node. Half mass of the segment between two adjacent nodes is added to the upper node and the remaining half added to the lower node. The equivalent rotational inertia with respect to the node at each floor is also calculated. The element stiffness matrix of the simplified model is formulated by the displacement method.

The dynamic characteristics obtained from the full model and the simplified are listed in Table 2. Their relative errors to compare the accuracy of two models are calculated by

$$E = \frac{|f_t - f_s|}{f_t} \times 100\% \quad (8)$$

where f_t and f_s are the frequencies of the full model and the simplified model, respectively. One can see that the dynamic characteristics of the two models are in very good agreement, in which the maximum frequency difference of the first 15 modes is only 0.17%.

4.2.3. Reduced FE model

Although the simplified model has fewer DOFs compared with full-order model, only translational DOFs could be considered for possible sensor installation in case study, as rotational DOFs are usually

Table 2. Dynamic characteristics of different numerical model.

Mode	Full-order FE model Frequency (Hz)	Reduced FE model					
		Simplified FE model		With antennary mast		Without antennary mast	
		Frequency (Hz)	Difference (%)	IIRS (Hz)	Difference (%)	IIRS (Hz)	Difference (%)
1	0.110	0.110	0.09	0.110	0.00	0.110	0.00
2	0.159	0.159	0.13	0.159	0.00	0.159	0.00
3	0.347	0.346	0.17	0.346	0.00	0.346	0.00
4	0.368	0.369	0.13	0.369	0.00	0.369	0.00
5	0.400	0.399	0.01	0.399	0.00	0.399	0.00
6	0.461	0.461	0.01	0.461	0.00	0.461	0.00
7	0.485	0.485	0.00	0.485	0.00	0.485	0.00
8	0.738	0.738	0.01	0.738	0.00	0.738	0.00
9	0.902	0.903	0.02	0.903	0.00	0.903	0.00
10	0.997	0.997	0.01	0.997	0.00	0.997	0.00
11	1.038	1.037	0.04	1.037	0.00	1.037	0.00
12	1.122	1.122	0.01	1.122	0.00	1.122	0.00
13	1.244	1.244	0.01	1.244	0.00	1.244	0.00
14	1.503	1.503	0.00	1.503	0.00	1.503	0.00
15	1.726	1.726	0.00	1.726	0.00	1.726	0.00

FE, finite element; IIRS, Iterated Improved Reduced System.

difficult to measure. Here, a thought of taking the horizontal DOF as the master DOF and rotational DOF as the slave DOF, and reducing the slave DOF by the model reduction is implemented. From the mathematical point of view, the model reduction may be considered as a kind of physical coordinate transformation. One of the oldest and most popular reduction methods is the Guyan or static reduction (Guyan, 1965). This method, although exact for the reduction of static problems, introduces large errors when applied to the reduction of dynamic problems. The various methods proposed as the modifications of Guyan reduction were developed, such as Kuhar (dynamic) Reduction (Kuhar and Stahle, 1974), Improved Reduced System (IRS) (O'Callahan, 1989) and Iterated Improved Reduced System (IIRS) (Friswell *et al.*, 1995). In this paper, the IIRS method is adopted because it converges more quickly and has higher accuracy.

Taking into account that the main tower is made of reinforced concrete interior tube and steel external tube, whereas the antennary mast is made of steel, the stiffness between them are quite different. Two cases are considered here: one is to retain the antenna (Figure 5(c)), and another is completely condensation off the DOFs of the antenna (Figure 5(d)). The results are shown in Table 1. It can be seen from the table that the eigenvalues obtained after 275 iterations (with antennary mast) and 1671 iterations (without antennary mast) by the IIRS method can be identical with those of the simplified FE model.

4.3. Optimization results and discussion

4.3.1. Sensor placement for whole tower (with antennary mast)

Figure 6 shows the MAC values of the initial placement (for the first three DOFs) determined by the QR factorization. As shown, the maximum off-diagonal element of the sensor placement determined by the QR factorization is close to the diagonal element, and there are many higher off-diagonal elements; thus, the result is not so good.

Figure 3(a) shows the variation curve of the maximum off-diagonal element for adding one more sensor to the initial placement. It is expected that the maximum MAC off-diagonal terms will decrease with more sensors included. This is really the case, more or less a decreasing trend as shown in the first segment of the curve in Figure 3(a) until 67 sensors are added. However, an interesting phenomenon that the maximum MAC off-diagonal terms become unexpectedly larger when more sensors are added further can be seen obviously in Figure 3(a) for having added more than 67 sensors.

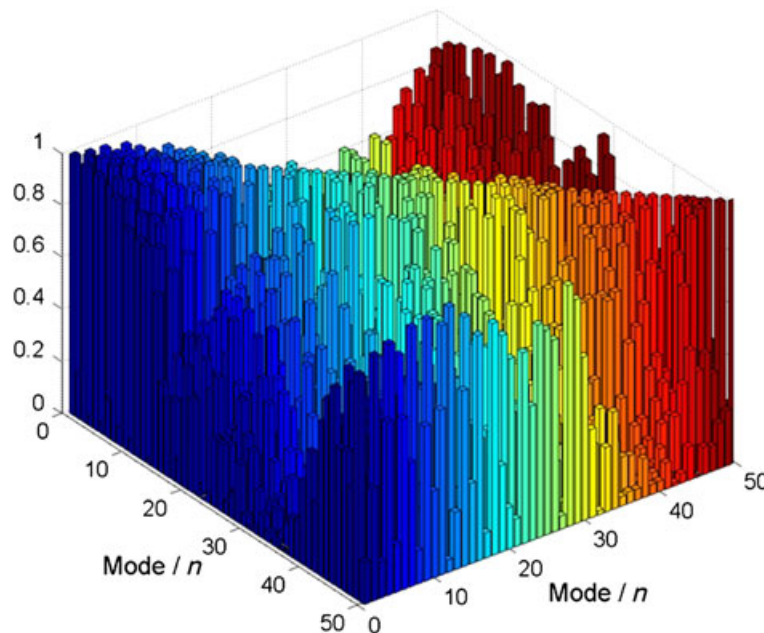


Figure 6. Modal assurance criterion value of the initial placement determined by the QR factorization.

In the same way, the maximum MAC off-diagonal values with decreasing number of sensors are depicted in Figure 3(b). It is supposed that the maximum MAC off-diagonal terms would increase when more sensors are excluded. This is really the increasing trend shown in Figure 3(b) when one observes the curve as a whole. However, the maximum MAC off-diagonal term becomes smaller when more sensors are excluded at the initial stage, even though they are expected to become larger. This is a similar phenomenon as observed in Figure 3(a). The reason for such increasing/decreasing contradiction is that a newly included or excluded sensor may conflict with other previously selected sensors. Mathematically speaking, the row vector determined at this newly included or excluded sensor position has strong linear relationship with the previous whole sensor set. This kind of uncommon phenomenon indicates that the combinational results provided by the SSP methods are more reasonable. From the two curves, it can be achieved that the lowest points are the value for adding 67 DOFs and deleting 4 DOFs, respectively. However, it is not economic due to high cost of the data acquisition systems (sensors and their supporting instruments). As shown from the two curves, the maximum off-diagonal element varies gently with 28 sensors. Therefore, 28 DOFs are selected as the sensor quantity, and the MAC values are shown in Figure 7. As shown from Figure 7, after adding 25 measurement points, the off-diagonal elements of the MAC matrix obtained by the SSP method are dramatically reduced, which means that the identical modal vector would be more distinguishable than the former one.

As known, the GGA has a number of parameters that are problem specific and needs to be explored and tuned so that the best algorithm performance is achieved. These parameters are the population size, leading population size and number of sudden change. To find out the most appropriate population size for minimal computation cost, various simulations with different population sizes have been first done, and a population size of 200 has been found to be adequate. In the simulation of the GGA process, the leading population size is defined as one quarter of the population size, and a relative large number 10 of sudden changes are selected to avoid redundant iteration. The genetic process will be stopped automatically if the sudden changes continuously happen 10 times. For the problem at hand, the size of the searching space is the number of DOFs on the reduced FE model excluding the constrained node. Excluding the one constrained node, there are 37 candidate nodes that could be used to place sensors. Because each node has two horizontal DOFs, 74 DOFs define the

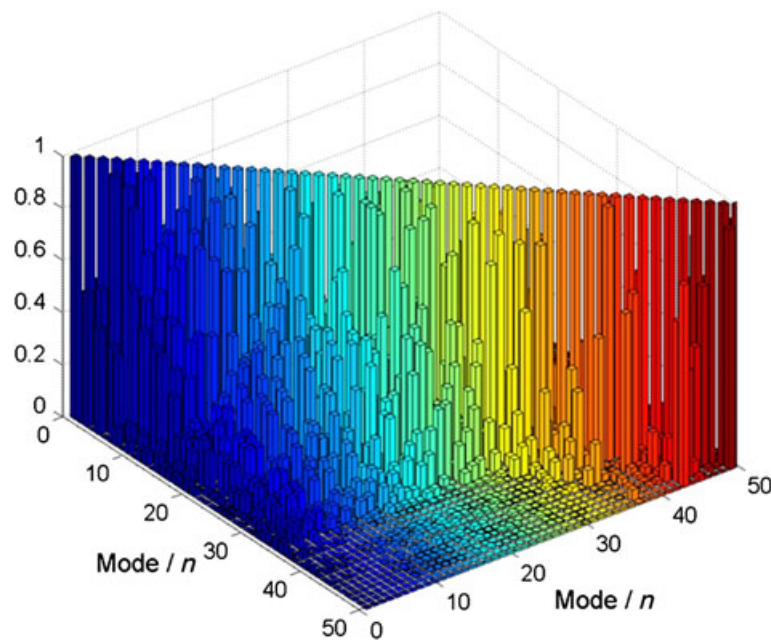


Figure 7. Modal assurance criterion value of selected 28 degrees of freedom obtained by the sequential sensor placement method.

searching space for the GGA process. It should be noticed that, due to the nature of the GA's method, the results are usually dependent on the randomly generated initial conditions, which means that the algorithm may converge to a different result in the parameter space. For the problem considered in this paper, the GGA process has been run for five times, and the best result is shown in Figure 3(c), where both the fitness progress of the best individual found by the algorithm as well as the average and worst fitness of the entire population at each generation are plotted. The optimization in the entire GGA population can be seen from the general decrease of the average population fitness, despite the numerous fluctuations caused by the search process through the genetic operators of crossover and mutation. It is obvious that all the best fitness values tend to a constant quickly, and the average fitness value steadily tends to the minimum fitness value along with increasing number of generations. It shows a good characteristic of convergence. The final sensor placement result of the GNTVT obtained is given in Table 3.

4.3.2. Sensor placement for main tower (without antennary mast)

Figure 8(a) shows the variation curve of the maximum off-diagonal element for adding one more sensor to the initial placement. Similarly, the maximum MAC off-diagonal values with decreasing number of sensors are depicted in Figure 8(b). From the two figures, it can be easily found that the curves are completely different from the first case (Section 4.3.1). As shown from the two curves, the maximum off-diagonal element varies gently with 20 sensors, i.e. there is a visible turning point. Therefore, 20 DOFs are selected as the sensor quantity. Figures 9 and 10 are the MAC values determined by the QR factorization and SSP method, respectively. As shown from Figure 10, after adding 20 measurement points, the off-diagonal elements of the MAC matrix obtained by the SSP method is also dramatically reduced. The final sensor placements for the main tower of the GNTVT obtained by the GGA are given in Table 4. The fitness progress of the best individual found by the algorithm as well as the average and worst fitness of the entire population at each generation are plotted in Figure 8(c). Compared with Table 3, the two results are significantly different. This may be due to the oversized local mode shape of the antennary mast that may cause certain influence on the sensor placement of the main tower with bigger stiffness. Thus, for the structures with stiffness mutation (like the main tower and antennary mast of GNTVT), the sensor placement had better to be considered for each part separately.

5. CONCLUSIONS

Large and complex civil structures are being placed in new and extreme conditions for extended periods. As a result, the need for robust and accurate health monitoring techniques continues to grow. An actual problem that should be solved by the engineer in the construction and implementation of an effective SHM system is how many sensors should be used and what is their optimal location to monitor the desired structural response. This paper has presented a hybrid method and developed a software package that could be used practically by civil engineers. With the case analysis, some conclusions and recommendations are summarized as follows:

- The hybrid method based on multiple optimization strategies is proposed in this study and is shown to be effective for determining the optimum number of sensors together with their best possible locations. Comparing the predictions with the FSSP and the BSSP methods, the results slightly differed from each other that may be depending on the sensor case considered. The combinational results provided by the SSP methods are recommended to give a better solution. The GGA is particularly effective in solving the combinatorial optimization problem such as the OSP problem.

Table 3. Sensor placement result of the Guangzhou New TV Tower (with antennary mast).

Sensor number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
DOFs X direction	1		3				9			12		14		15		17			19	23	26		27	28	31	34	35		
Y direction		1		4	5	6		9	10		12		14		16		17	18					26						35

DOFs, degrees of freedom.

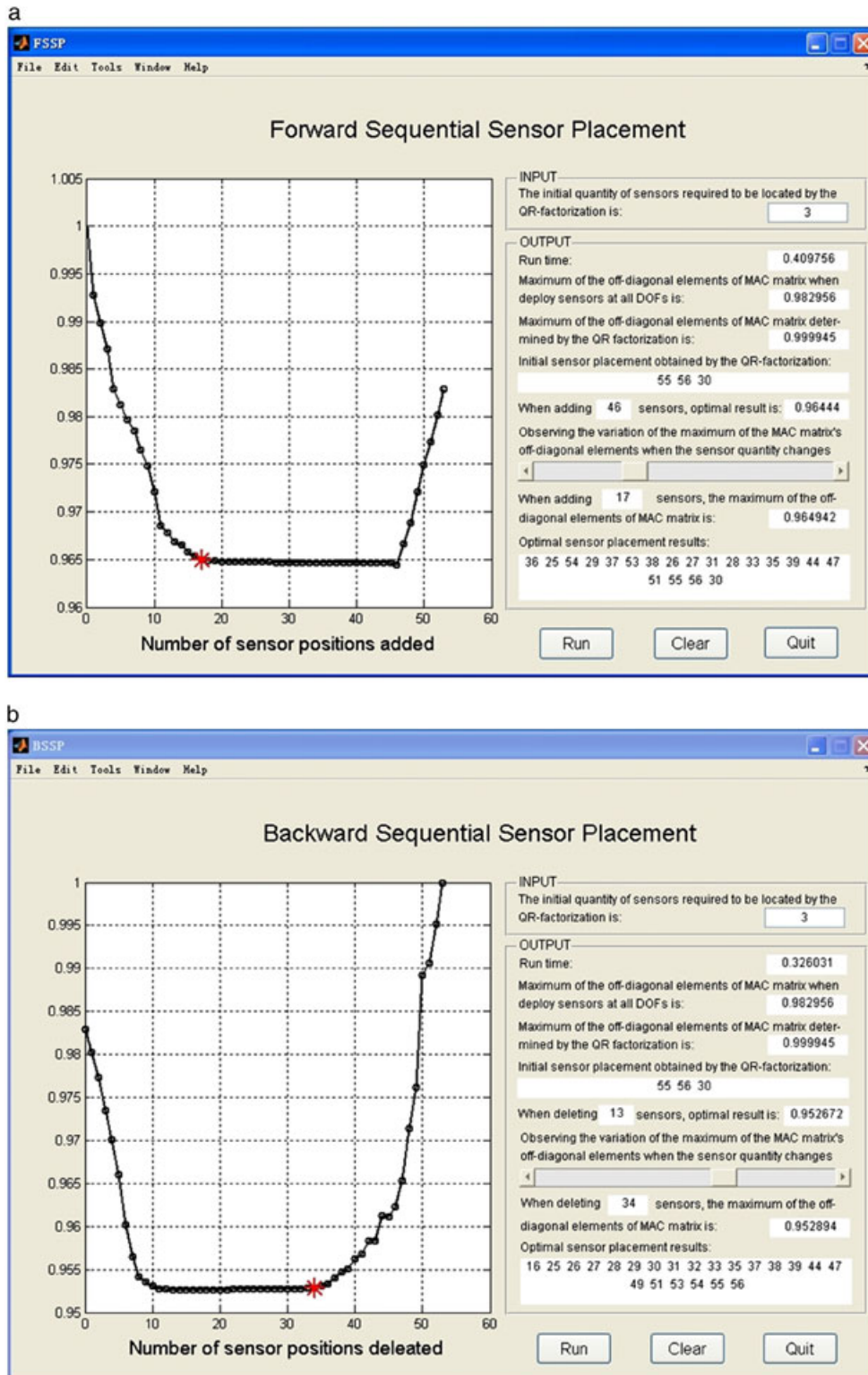


Figure 8. Calculation results for the main tower of the Guangzhou New TV Tower: (a) forward sequential sensor placement; (b) backward sequential sensor placement; (c) generalized genetic algorithm.

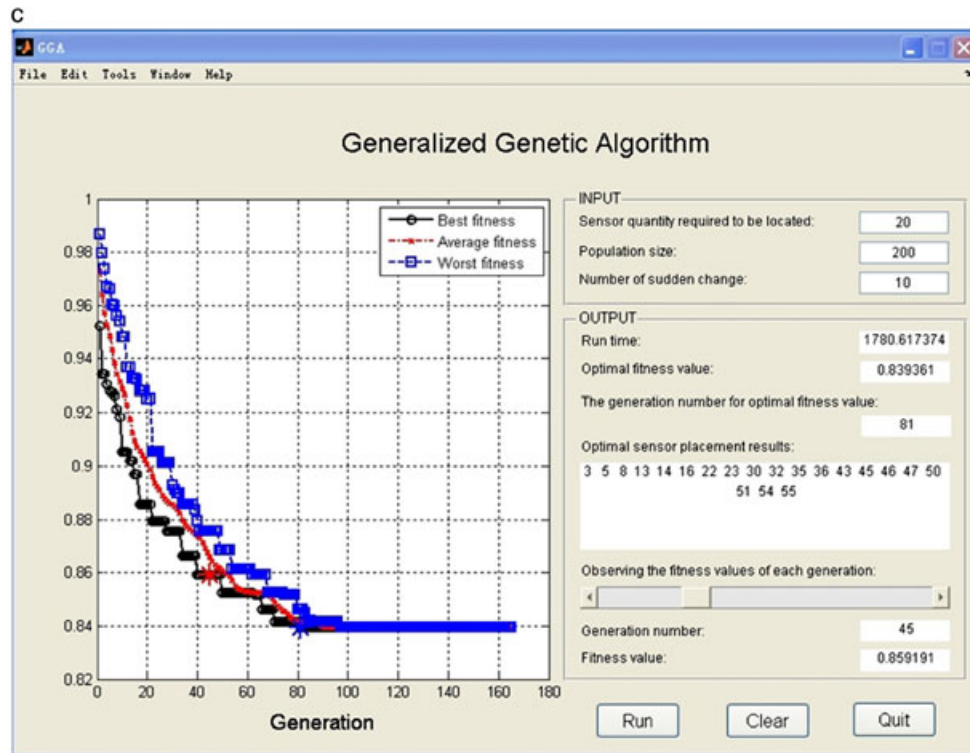


Figure 8. (Continued)

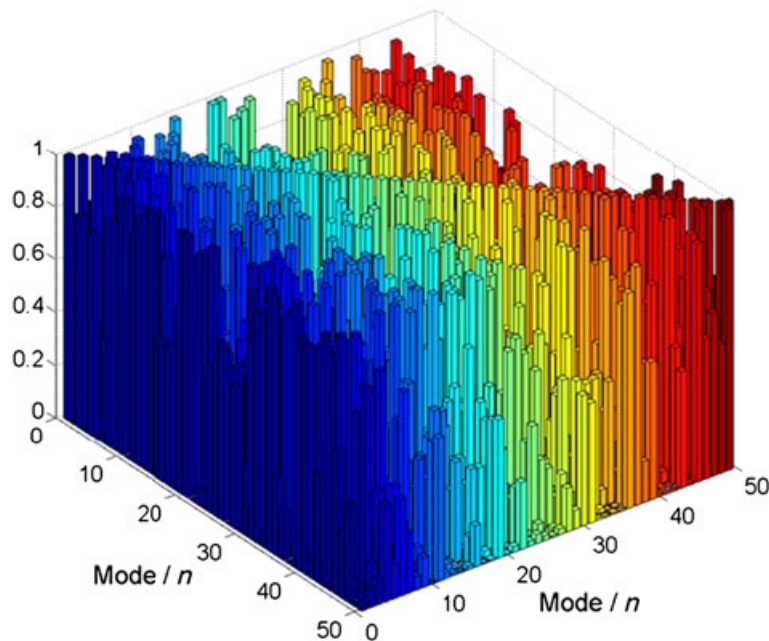


Figure 9. Modal assurance criterion value of the initial placement determined by the QR factorization.

Due to the random nature of the initial population used in the GGA, the proposed hybrid optimization algorithm is effective of determining global minima by running the GGA algorithm several times and storing the optimal solution of each run into an optimal set of solutions.

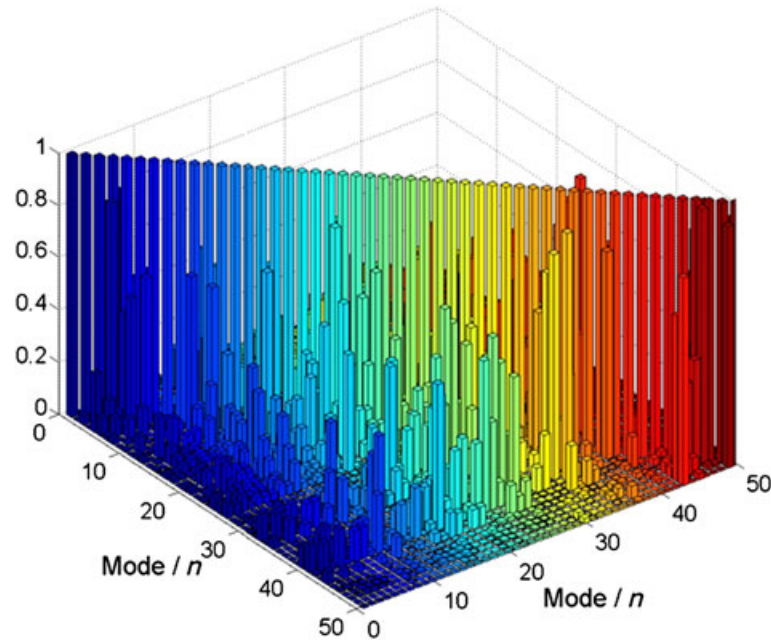


Figure 10. Modal assurance criterion value of selected 28 degrees of freedom obtained by the sequential sensor placement method.

Table 4. Sensor placements of the Guangzhou New TV Tower (without antennary mast).

Sensor number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
DOFs <i>X</i> direction	2	3		7				12			18		22	23		24		26		28
DOFs <i>Y</i> direction			4		7	8	11		15	16		18			23		25			27

DOFs, degrees of freedom.

- Working with multiple programs and computer platforms for processing and analysis, to some extent, may be very tedious for civil engineers. To overcome this problem, a flexible toolbox called the 'OSPS Toolbox' in MATLAB is developed. A simple GUI is used to navigate the toolbox and thus allows a user not familiar with programming languages to apply the implemented methods in an easy way, and it also gives a straightforward possibility to visualize the obtained results. On the other hand, the toolbox is fully extensible, allowing new modules to be added by those familiar with the MATLAB programming language. Although the software does not require a highly experienced user, a basic knowledge on the underlying methods is advisable in order to successfully interpret the results. As an illustrative example, the toolbox is used to the world tallest TV tower (GNTVT) for scheme selection of the OSP. The evaluation process successfully shows that the toolbox is fairly convenient, very useful and quite efficient.
- The thought of taking the horizontal DOF as the master DOF and rotational DOF as the slave DOF, and reducing the slave DOF by the model reduction is proposed. This change as well as model simplification in optimization process may result in a dramatic reduction in the required computational effort caused by tens of thousands of DOF.
- For the structures with stiffness mutation (like the main tower and antennary mast of GNTVT), the sensor placement had better to be considered for each part separately. On one hand, the critical failure limits of them may be different, (e.g., the main tower is concrete structure, while the antennary is steel structure; the allowable values of deformation are different). On the other hand, the vibration response may be larger for the structures with smaller stiffness. For the optimization criterions which only consider the mode shape (such as, MAC criterion), the oversized local mode shape may have certain influence on the sensor placement of the structures with bigger stiffness.

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REFERENCES

- Carne TG, Dohmann CR. 1995. A modal test design strategy for modal correlation. In *Proceedings of the 13th International Modal Analysis Conference*. Union College, Schenectady: New York; 927–933.
- Chang FK, Markmiller JFC. 2006. A new look in design of intelligent structures with SHM. In *Proceeding of 3rd European Workshop: Structural Health Monitoring*, Granada, Spain, July.
- Chung YT, Moore JD. 1993. On-orbit sensor placement and system identification of space station with limited instrumentations. In *Proceedings of the 11th International Modal Analysis Conference*, Kissimmee, Orlando, USA, February 1993.
- Dong C. 1998. Generalized genetic algorithm. *Exploration of Nature* **63**(17): 33–37.
- Field RVJ, Grogoriu M. 2006. Optimal design of sensor networks for vehicle detection, classification and monitoring. *Probabilistic Engineering Mechanics* **21**(4): 305–316.
- Friswell MI, Garvey SD, Penny JET. 1995. Model reduction using dynamic and iterated IRS techniques. *Journal of Sound and Vibration*, **186**(2): 311–323.
- Gu M, Huang P, Zhou XY, Zhu LD, Pan HM. 2009. Wind tunnel force balance test and wind-induced response of the Guangzhou New TV Tower structure I: wind tunnel test. *China Civil Engineering Journal* **42**(7): 8–13.
- Guo YL, Wang YH, Liu LY, Lin B, Pan HM, Liang S. 2010. Experimental studies on multi-column out-plane buckling in bottom open-space region of the Guangzhou New TV Tower. *Journal of Building Structures* **31**(1): 78–86.
- Guyan RJ. 1965. Reduction of stiffness and mass matrices. *American Institute of Aeronautics and Astronautics Journal* **3**(2): 380.
- Heredia-Zavoni E, Montes-Iturrizaga R, Esteve L. 1999. Optimal instrumentation of structures on flexible base for system identification. *Earthquake Engineering and Structure Dynamics* **28**(12): 1471–1482.
- Holland JH. 1975. *Adaptation in natural and artificial systems*. University of Michigan Press: Ann Arbor.
- Kammer DC. 1991. Sensor placement for on-orbit modal identification and correlation of large space structures. *Journal of Guidance, Control Dynamics* **14**(2): 251–259.
- Kammer DC, Tinker ML. 2004. Optimal placement of triaxial accelerometers for modal vibration tests. *Mechanical Engineering and Signal Processing* **18**: 29–41.
- Kistera G, Badcocka RA, Gebremichaelb YM, Boyle WJO, Grattan KTV, Fernando GF, Canning L. 2007. Monitoring of an all-composite bridge using Bragg grating sensors. *Construction and Building Materials* **21**(7): 1599–1604.
- Kuhar EJ, Stahle CV. 1974. Dynamic transformation method for modal synthesis. *American Institute of Aeronautics and Astronautics Journal* **12**(5): 672–678.
- Li HN, Li DS, Song GB. 2004. Recent applications of fiber optic sensors to health monitoring in civil engineering. *Engineering Structures* **26**(11): 1647–1657.
- Li DS, Li HN, Fritzen CP. 2007a. The connection between effective independence and modal kinetic energy methods for sensor placement. *Journal of Sound and Vibration* **305**(4–5): 945–955.
- Li DS, Li HN, Fritzen CP. 2007b. Representative least squares method for sensor placement. *Proceedings of the 3rd International Conference on Structural Health Monitoring and Intelligent Infrastructure*, Vancouver, Canada, November.
- Li DS, Fritzen CP, Li HN. 2008. Extended MinMAC algorithm and comparison of sensor placement methods. *IMAC-XXVI*, Orlando, FL, USA, 4–7 February.
- Lin YSF, Chiu PK. 2005. A near optimal sensor placement algorithm to achieve complete coverage/discrimination in sensor networks. *IEEE Communications Letters* **9**(1): 43–45.
- Lin W, Ni YQ, Xia Y, Chen WH. 2010. Field measurement data and a reduced order finite element model for Task I of the SHM benchmark problem for high rise structures. *Proceedings of the 5th World Conference on Structural Control and Monitoring*, Tokyo, Japan, 12–14 July 2010.
- Liu C, Tasker FA. 1996. Sensor placement for time-domain modal parameter estimation. *Journal of Guidance, Control, and Dynamics* **19**(6): 1349–1356.
- Liu LY, Guo YL, Wang YH. 2009. Influence of the horizontal bracings on the stability of the exterior steel frame in the waist region of the Guangzhou New TV Tower. *China Civil Engineering Journal* **42**(5): 61–68.
- Majumder M, Gangopadhyay TK, Chakraborty AK, Dasgupta K, Bhattacharya DK. 2008. Fibre Bragg gratings in structural health monitoring-present status and applications. *Sensors and Actuators A: Physical* **147**(1): 150–164.
- MATLAB. 2008. The MathWorks, Inc. Natick, MA (USA), <http://www.mathworks.com>.
- Meo M, Zumpano G. 2005. On the optimal sensor placement techniques for a bridge structure. *Engineering Structures* **27**(10): 1488–1497.
- Ni YQ, Xia Y, Liao WY, Ko JM. 2009. Technology innovation in developing the structural health monitoring system for Guangzhou New TV Tower. *Structural Control and Health Monitoring* **16**(1): 73–98.
- O'Callahan JC. 1989. A procedure for an improved reduced system IRS model. *Proceedings of the 6th International Modal Analysis Conference*, Las Vegas, Jan. 1989; 471–479.

- Pan HM, Zhou FL, Liang S. 2008. Shaking table test for the structural model of Guangzhou New TV Tower. *Engineering Mechanics* **25**(11): 78–85.
- Papadimitriou C. 2004. Optimal sensor placement methodology for parametric identification of structural systems. *Journal of Sound and Vibration* **278**: 923–947.
- Salama M, Rose T, Garba J. 1987. Optimal placement of excitations and sensors for verification of large dynamical systems. *Proceedings of the 28th Structures, Structural Dynamics, and Materials Conference*, Monterey, California, USA, April 1987.
- Tan P, Bu GX, Zhou FL. 2009. Study on wind-resistant dynamic reliability of TMD with limited spacing. *Journal of Vibration and Shock* **28**(6): 42–45.
- Wenzel H. 2009. *Health monitoring of bridges*. John Wiley and Sons Ltd.: Hoboken, NJ.

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